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MOMENTUM AND ENERGY RELAXATION BY AN ELECTRON IN A CRYSTAL III.--ETC(U)  
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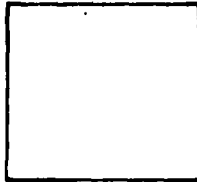
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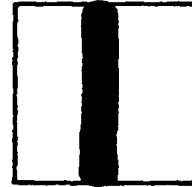
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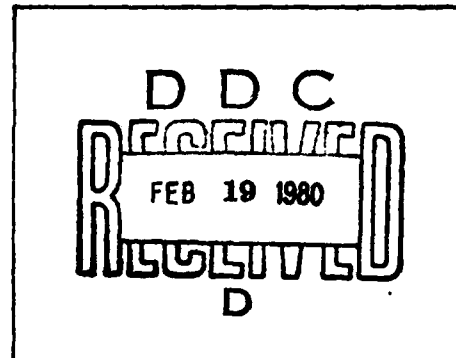
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## FOREIGN TECHNOLOGY DIVISION



MOMENTUM AND ENERGY RELAXATION BY AN ELECTRON IN A CRYSTAL  
III. COMPOUND SCATTERING BY OPTICAL PHONONS AT LOW TEMPERATURES

By

P. A. Kazlauskas, I. B. Levinson, G. E. Mazhuolite



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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b><i>А а</i></b>	A, a	Р р	<b><i>Р р</i></b>	R, r
Б б	<b><i>Б б</i></b>	B, b	С с	<b><i>С с</i></b>	S, s
В в	<b><i>В в</i></b>	V, v	Т т	<b><i>Т т</i></b>	T, t
Г г	<b><i>Г г</i></b>	G, g	У у	<b><i>У у</i></b>	U, u
Д д	<b><i>Д д</i></b>	D, d	Ф ф	<b><i>Ф ф</i></b>	F, f
Е е	<b><i>Е е</i></b>	Ye, ye; E, e*	Х х	<b><i>Х х</i></b>	Kh, kh
Ж ж	<b><i>Ж ж</i></b>	Zh, zh	Ц ц	<b><i>Ц ц</i></b>	Ts, ts
З э	<b><i>З э</i></b>	Z, z	Ч ч	<b><i>Ч ч</i></b>	Ch, ch
И и	<b><i>И и</i></b>	I, i	Ш ш	<b><i>Ш ш</i></b>	Sh, sh
Й й	<b><i>Й й</i></b>	Y, y	Щ щ	<b><i>Щ щ</i></b>	Shch, shch
К к	<b><i>К к</i></b>	K, k	Ъ ъ	<b><i>Ъ ъ</i></b>	"
Л л	<b><i>Л л</i></b>	L, l	Ы ы	<b><i>Ы ы</i></b>	Y, y
М м	<b><i>М м</i></b>	M, m	Ь ь	<b><i>Ь ь</i></b>	'
Н н	<b><i>Н н</i></b>	N, n	Э э	<b><i>Э э</i></b>	E, e
О о	<b><i>О о</i></b>	O, o	Ю ю	<b><i>Ю ю</i></b>	Yu, yu
П п	<b><i>П п</i></b>	P, p	Я я	<b><i>Я я</i></b>	Ya, ya

\*ye initially, after vowels, and after ъ, ь; e elsewhere.  
When written as ë in Russian, transliterate as yë or ë.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian      English

rot      curl  
lg      log

0372, gw

MOMENTUM AND ENERGY RELAXATION BY AN ELECTRON IN A CRYSTAL

III. COMPOUND SCATTERING BY OPTICAL PHONONS AT LOW TEMPERATURES

P. A. Kazlauskas, I. B. Levinson, G. E. Mazhuolite

(Received 10 Dec. 1965)

The coefficients of the Fokker-Planck equation are calculated during low-temperature scattering by optical phonons for the random law of dispersion and for electron energies up to the energy of an optical phonon.

1. Introduction. The probability of compound scattering.

Under conditions when the lattice temperature  $kT$  is much less than the maximum frequency of optical phonons  $\hbar\omega_0$ , the probability of emission of a phonon is much greater than the probability of absorption of the phonon. Therefore absorption is accompanied by "instantaneous re-emission" and these two sequential processes may be examined as a single act of compound scattering [1, 2]. Thanks to the small dispersion of optical phonons the compound scattering is almost elastic and precisely this circumstance makes it possible to convert the integral collision term of the kinetic equation from Boltzmann's form to the differential form of Fokker-Planck. While completing this procedure B. Davydov and N. Shamuskevich [2] assumed that in a considerable area of energies  $\epsilon \ll \hbar\omega_0$ , the anisotropy of the distribution function is also small, i.e.,

$$(1.1) \quad f_0 = f_0(\epsilon) + f_1(\epsilon) \cos \vartheta, \quad f_1(\epsilon) \ll f_0(\epsilon).$$

Here  $p$  is the pulse of the electron,  $\vartheta$  is its angle with the direction of the electrical field. These assumptions in fact are not significant; in [3] it was shown that the concept of compound scattering is limited only by the condition  $\epsilon \ll \hbar\omega_0$ , when the scattering begins necessarily from absorption and that it may be described by the probability of compound scattering:

$$(1.2) \quad \hat{W}(p, p') = \int (dp'') \epsilon_0^+(p'') W^-(p, p'') W^+(p'', p').$$

Here  $p$  and  $p'$  are the initial and final states with energies  $\epsilon$ ,  $\epsilon' < \omega_0$ .  $p''$  is an intermediate state with an energy of  $\epsilon' > \omega_0$ . The lifetime in this intermediate state (due to emission) is

$$(1.3) \quad \frac{1}{\tau_0^+(p'')} = \int (dp') W^+(p'', p').$$

Factor  $W$  describes the absorption transition from  $p$  to  $p''$ , and  $W^+ \tau_0^+$  is the conditional probability that emission of the phonon will lead to the transition from  $p''$  to the final point  $p'$  which is of interest to us.

Integrating (1.2) with respect to  $p'$  it is easy to check that the lifetime of state  $p$  relative to compound scattering  $\frac{1}{\tau_0}(p)$  coincides with the lifetime relative to initial scattering  $\tau_0^-(p)$ : this is physically clear because  $1/\tau_0$  is contributed to equally by all processes of phonon absorption independently of what happens with this electron following absorption. However, for other characteristic times such a coincidence does not occur.

Subsequently we shall use an isotropic model [4, 5]. Expanding the compound and regular probability through Legendre polynomials [2.7] 1, for the momentums we find



$$(1.4) \quad \hat{W}_I(\epsilon, \epsilon') = \int_{\epsilon_0}^{\infty} d\epsilon'' \epsilon_0^+(\epsilon'') W_I^-(\epsilon, \epsilon'') W_I^+(\epsilon'', \epsilon').$$

It is not difficult to check that if the initial probability  $W$  satisfies the principle of detailed scattering, then the latter is automatically satisfied by the compound probability  $\hat{W}$ . Let us note one more obvious circumstance: concepts of compound scattering and its probability (1.2) refer not only to polarization scattering by optical phonons PO, examined in (2), but also to differential scattering DO.

## 2. Characteristics of compound scattering.

Below we calculate values which describe compound scattering of an electron. The sign  $\hat{\phantom{x}}$ , which indicates the compound nature of the scattering, is omitted for brevity.

We introduce the following designations:

minimum and maximum momentums of absorbed phonons

$$(2.1) \quad \begin{aligned} q_1^-(\epsilon) &= p(\epsilon + \hbar\omega_0) - p(\epsilon), \\ q_2^-(\epsilon) &= p(\epsilon + \hbar\omega_0) + p(\epsilon), \end{aligned}$$

momentums of matrix elements of interaction

$$(2.2) \quad B_n(\varepsilon) = \int_{q_1^-(\varepsilon)}^{q_n^-(\varepsilon)} dq \, q^n B(q),$$

auxiliary function

$$(2.3) \quad \Lambda(\varepsilon) = \ln \frac{q_n^-(\varepsilon)}{q_1^-(\varepsilon)},$$

number of phonons (2.4)

$$N_0 = \exp \left\{ - \frac{\hbar \omega_0}{kT} \right\},$$

characteristic momentum and velocity

$$(2.5) \quad p_0 = p(\hbar \omega_0), \quad v_0 = v(\hbar \omega_0).$$

All other unstipulated designations follow I and II.

During calculation of the lifetime  $\tau_0$  and the longitudinal relaxation time of momentum  $\tau_{||} = \tau$  the dispersion of optical phonons may be disregarded, i.e., compound scattering may be considered elastic. Using the probability of transition in the form (2.4) I, calculating the momentums of compound probability with (2.9) I and

(1.4) and substituting everything in (3.2) I and (3.5) I, we find the lifetime:

$$(2.6) \quad \frac{1}{\tau_0(z)} = \frac{2\pi}{\hbar} N_0 2\pi \frac{p(z+\hbar\omega_0)}{p(z) v(z+\hbar\omega_0)} B_1(z),$$

and the relaxation time:

$$(2.7) \quad \frac{1}{\tau(z)} = \frac{1}{\tau_0(z)} \left[ 1 - \left( \coth \Lambda(z) - \frac{1}{2p(z) p(z+\hbar\omega_0)} \frac{B_2(z)}{B_1(z)} \right)^2 \right].$$

During calculation of the coefficient of energy diffusion  $D$  one should take into account the dispersion of optical phonons which is accepted in the following form:

$$(2.8) \quad \hbar\omega(q) = \hbar\omega_0 - \frac{q^2}{2M}.$$

where  $M$  is the order of mass of the nucleus. Considering the dispersion term in the law of preservation of energy and expanding the corresponding  $\delta$ -functions in the probabilities of transitions  $W^+$  and  $W^-$ , entering into (1.2), to the second order with respect to  $M^{-1}$  inclusively, we obtain

$$(2.9) \quad D(z) = \frac{1}{\tau_0(z)} \frac{1}{4M^2} \left[ \frac{B_2(z)}{B_1(z)} - \frac{B_2^2(z)}{B_1^2(z)} \right].$$

Compound scattering is elastic everywhere and satisfies the principle of detailed equilibrium; therefore the power of energy losses  $Q$  can be calculated by direct differentiation using (2.9) and (1.32) II.

Using the matrix elements  $B(q)$  from I and calculating momentum  $B_s(\varepsilon)$ , it is possible to find the following expressions for specific mechanisms of interaction:

DO

$$\frac{1}{\tau_0(\varepsilon)} = \frac{1}{\tau(\varepsilon)} = \frac{2\pi}{\hbar} B_0 N_0 4\pi \frac{p^2(\varepsilon + \hbar\omega_0)}{v(\varepsilon + \hbar\omega_0)}, \quad (2.10)$$

$$D(\varepsilon) = \frac{1}{\tau_0(\varepsilon)} \frac{p^2(\varepsilon) p^2(\varepsilon + \hbar\omega_0)}{3M^2}, \quad (2.11)$$

PO

$$\frac{1}{\tau_0(\varepsilon)} = \frac{2\pi}{\hbar} B_0 N_0 2\pi \frac{p(\varepsilon + \hbar\omega_0)}{p(\varepsilon) v(\varepsilon + \hbar\omega_0)} \Lambda(\varepsilon), \quad (2.12)$$

$$\frac{1}{\tau(\varepsilon)} = \frac{1}{\tau_0(\varepsilon)} \left[ 1 - \left( \text{cth } \Lambda(\varepsilon) - \frac{1}{\Lambda(\varepsilon)} \right)^2 \right], \quad (2.13)$$

$$D(\varepsilon) = \frac{1}{\tau_0(\varepsilon)} \frac{p^2(\varepsilon) p^2(\varepsilon + \hbar\omega_0)}{M^2} \frac{1}{\Lambda(\varepsilon)} \left( \text{cth } \Lambda(\varepsilon) - \frac{1}{\Lambda(\varepsilon)} \right). \quad (2.14)$$

For a parabolic zone the results are simplified somewhat and

$$(2.15) \quad \Lambda(\varepsilon) = 2 \text{Arsh} \left( \frac{\varepsilon}{\hbar\omega_0} \right)^{\frac{1}{2}}.$$

The relaxation time  $\tau$  for a parabolic zone and the mechanism PO were calculated using methods of resonance scattering [6]. Disregarding of

the virtual transitions leads, in [6], to a formula different from (2.13) by the supplementary factor 2/3 in the right part, which, as was pointed out in [7], is the result of error in integration.

### 3. Low energies.

The obtained formulas are simplified significantly for low energies  $\varepsilon \ll \hbar\omega_0$ . Let us note that in this case

$$(3.1) \quad \Lambda(\varepsilon) = 2 \frac{p(\varepsilon)}{p_0} \ll 1,$$

$$(3.2) \quad \operatorname{cth} \Lambda(\varepsilon) - \frac{1}{\Lambda(\varepsilon)} = \frac{2}{3} \frac{p(\varepsilon)}{p_0} \ll 1.$$

Using these formulas we find that for low energies

$$(3.3) \quad \frac{1}{\tau(\varepsilon)} = \frac{1}{\tau_0(\varepsilon)} = \frac{2\pi}{\hbar} B_0 N_0 4\pi \frac{p_0^3}{v_0} \quad (DO),$$

$$(3.4) \quad = \frac{2\pi}{\hbar} B_0 N_0 4\pi \frac{1}{v_0} \quad (PO),$$

$$(3.5) \quad D(\varepsilon) = \frac{1}{\tau_0(\varepsilon)} \frac{1}{3} \frac{p^2(\varepsilon) p_0^3}{M^2}.$$

Times  $\tau$  and  $\tau_0$  in this case turn out to be independent of  $\varepsilon$  for any law of dispersion. With the parabolic law of dispersion these results coincide with those obtained in [2]. In this case it is also

convenient to introduce the relaxation time  $\tilde{\tau}$  and fluctuation time  $\bar{\tau}$  of energy

$$(3.7) \quad \begin{aligned} \frac{1}{\tilde{\tau}(\epsilon)} &= \frac{1}{\tau_0} \frac{4}{3} \frac{\hbar\omega_0}{kT} \left(\frac{m}{M}\right)^2, & \epsilon^* &= \frac{3}{2} kT \\ \frac{1}{\bar{\tau}(\epsilon)} &= \frac{1}{\tau_0} \frac{4}{3} \frac{\hbar\omega_0}{\epsilon} \left(\frac{m}{M}\right)^2. \end{aligned}$$

Here  $\epsilon^*$  is the energy with which  $Q(\epsilon)$  changes sign. From (3.6) it is evident that inelasticity of compound scattering by optical phonons is determined by the parameter

$$(3.8) \quad \bar{\delta} = \frac{4}{3} \frac{\hbar\omega_0}{kT} \left(\frac{m}{M}\right)^2.$$

Let us note additionally that the use of formulas obtained for  $\epsilon \ll \hbar\omega_0$  in the entire interval up to  $\epsilon = \hbar\omega_0$ , gives correct results with respect to the order of magnitude.

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### Summary

When lattice temperature  $kT$  and electron energy  $\varepsilon$  is lower than optical phonon energy  $\hbar\omega_0$ , the absorption of a phonon is followed by its immediate re-emission. These two scattering acts may be treated as a compound scattering, whose probability  $\tilde{W}$  is given by (1.2) in terms of ordinary scattering probability  $W$ .

General formulae are: (2.6) — life time, (2.7) — momentum relaxation time (longitudinal), (2.9) — energy diffusion coefficient; for special mechanisms: (2.10) and (2.11) — deformation scattering, (2.12) — (2.14) — polarization scattering.

For energies  $\varepsilon \ll \hbar\omega_0$  (3.1) — (3.5) are valid. At such energies and parabolic band the energy relaxation and fluctuation times, given by (3.6) and (3.7) respectively may be introduced. The elasticity of compound scattering is given by (3.8).

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